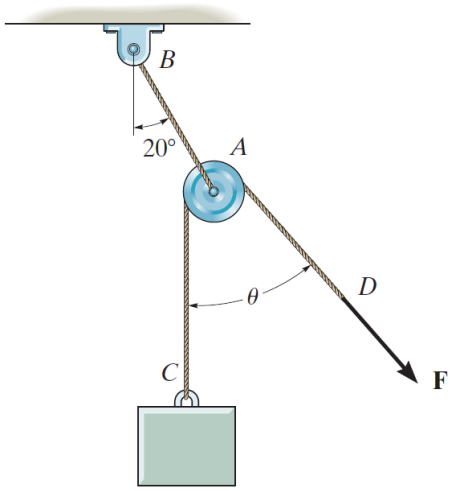
Problem 3-10

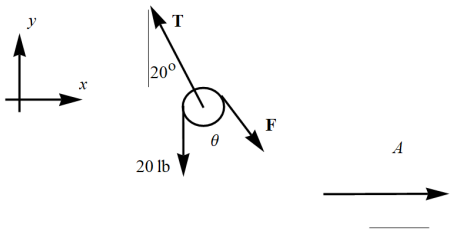
The block has a weight of 20 lb and is being hoisted at uniform velocity. Determine the angle θ for equilibrium and the force in cord AB.



Probs. 3–10/11

Solution

Draw a free-body diagram for the pulley.



In order for the block to be hoisted at constant velocity, the sum of the forces in each direction must be zero.

$$\sum F_x = 0: \qquad F\sin\theta - T\sin 20^\circ = 0$$

$$\sum F_y = 0: \qquad T\cos 20^\circ - F\cos\theta - 20 = 0$$

Because the pulley is frictionless, the tension in cord CAD is the same everywhere: $F = 20$ lb.

$$20\sin\theta - T\sin 20^\circ = 0 \tag{1}$$

$$T\cos 20^\circ - 20\cos\theta - 20 = 0 \tag{2}$$

Solve equation (1) for T

$$T = \frac{20\sin\theta}{\sin 20^\circ}$$

and substitute it into equation (2).

$$\left(\frac{20\sin\theta}{\sin 20^\circ}\right)\cos 20^\circ - 20\cos\theta - 20 = 0$$

$$\cot 20^\circ \sin\theta - \cos\theta - 1 = 0$$

$$\cot 20^\circ \sin\theta - 1 = \cos\theta$$

$$\cot 20^\circ \sin\theta - 1 = \sqrt{1-\sin^2\theta}$$

$$\cot^2 20^\circ \sin^2\theta - 2\cot 20^\circ \sin\theta + 1 = 1 - \sin^2\theta$$

$$(\cot^2 20^\circ + 1)\sin^2\theta - 2\cot 20^\circ \sin\theta = 0$$

$$\csc^2 20^\circ \sin^2\theta - 2\cot 20^\circ \sin\theta = 0$$

$$(\csc^2 20^\circ \sin\theta - 2\cot 20^\circ)\sin\theta = 0$$

$$\csc^2 20^\circ \sin\theta - 2\cot 20^\circ = 0 \quad \text{or} \quad \sin\theta = 0$$

$$\sin\theta = \frac{2\cot 20^\circ}{\csc^2 20^\circ} = 2\cos 20^\circ \sin 20^\circ = \sin 40^\circ \quad \text{or} \quad \sin\theta = 0$$

$$\theta = 40^\circ \quad \text{or} \quad \cancel{\theta = 0^\circ}$$

Substitute this nonzero value for θ into the formula for T.

$$T = \frac{20\sin\theta}{\sin 20^\circ} \approx 37.6 \text{ lb}$$